

# On the Determination of Minimum Fluidization Velocity by the Method of Yang et al.

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## Introduction

Recently Yang et al. (1985) have proposed a general methodology for determining minimum fluidizing velocity,  $U_{mf}$ , based on the work of Barnea and Mizrahi (1973) and Barnea and Mednick (1975). These formulations are rigorously correct only for spherical particles, and the procedure for determining  $U_{mf}$  at high temperatures and high pressures from the knowledge of the bed voidage at minimum fluidization,  $\epsilon_{mf}$ , at ambient conditions is briefly described in this note. A log-log plot is generated of  $(Re^2 C_{D,\epsilon})_{mf}^{1/3}$  vs.  $(Re_\epsilon/C_{D,\epsilon})_{mf}^{1/3}$  for the known value of  $\epsilon_{mf}$ . Here,

$$(Re_\epsilon^2 C_{D,\epsilon})_{mf}^{1/3} = (Re C_D)_{mf}^{1/3} \left\{ \frac{\epsilon_{mf}^{1/3}}{[1 + (1 - \epsilon_{mf})^{1/3}]^{1/3} \exp [10 (1 - \epsilon_{mf})/9 \epsilon_{mf}]} \right\}, \quad (1)$$

and

$$\left( \frac{Re_\epsilon}{C_{D,\epsilon}} \right)_{mf}^{1/3} = \left( \frac{Re}{C_D} \right)_{mf}^{1/3} \left\{ \frac{[1 + (1 - \epsilon_{mf})^{1/3}]^{1/3}}{\epsilon_{mf}^{4/3} \exp [5 (1 - \epsilon_{mf})/9 \epsilon_{mf}]} \right\}. \quad (2)$$

Further, the various correlations given by Clift et al. (1978) for single spherical particles are used to determine  $C_D$  as a function of  $Re$ . In the application of this procedure, Yang et al. assume that  $\epsilon_{mf}$  does not change with temperature and pressure. Hence, at the desired operating conditions,  $(Re_\epsilon^2 C_{D,\epsilon})_{mf}^{1/3}$  is computed from Eq. 1 where

$$(Re^2 C_D)_{mf}^{1/3} = \left( \frac{4Ar}{3} \right)^{1/3} = \left\{ \frac{d_p}{[3\mu^2/4g\rho_s(\rho_s - \rho_g)]^{1/3}} \right\}. \quad (3)$$

The corresponding value of  $(Re_\epsilon/C_{D,\epsilon})_{mf}^{1/3}$  is then read from the

plot, and using Eq. 2 and recalling that

$$\left( \frac{Re}{C_D} \right)_{mf}^{1/3} = \left\{ \frac{U_{mf}}{[4g\mu(\rho_s - \rho_g)/3\rho_g^2]^{1/3}} \right\}, \quad (4)$$

$U_{mf}$  is computed. Yang et al. found that this method led to good estimates of  $U_{mf}$  values for systems at ambient temperature and high pressures, and also for systems at ambient pressure and high temperatures. Most of these systems involved nonspherical particles of a size range.

## Computation of $U_{mf}$ and Discussion

We applied the generalized methodology of Yang et al. to the data of Saxena and Vogel (1977) on nonspherical dolomite particles of the size range 88–1,410  $\mu\text{m}$  and for temperatures and pressures in the ranges, 291–700 K and 179–834 kPa, respectively. In view of the wide size range of these particles, and the fact that these data correspond to moderately high temperature and pressure conditions, we have attempted to evaluate the validity of the methodology of Yang et al. on the basis of these data. The results of our computations for the two series of data are shown in Figure 1. The agreement between the predicted and experimental results is excellent, which suggests that for all practical purposes, the scheme of Yang et al. appears to be reliable. The fact that the formulation is essentially for spherical particles does not seem to matter very much.

Another startling implication of the above analysis that  $\epsilon_{mf}$  does not depend on pressure and temperature for a given system needs to be examined rather carefully in view of the recent results of Botterill et al. (1982) and Lucas et al. (1985). Patti-pati and Wen (1982) have claimed that the observed dependence of  $\epsilon_{mf}$  on bed temperature reported by Botterill et al. is due

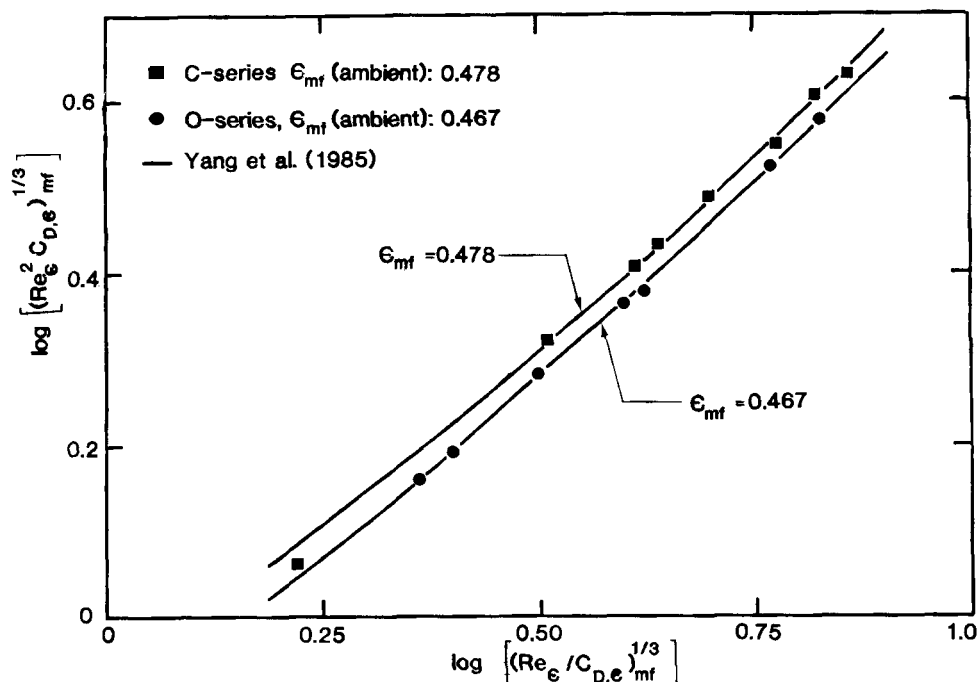


Figure 1. Comparison of experimental  $Re_{mf}$  values of Saxena and Vogel (1977) with predictions based on method of Yang et al. (1985).

to the fluidizing air not being preheated to the bed temperature before introduction in the bed. However, the measurements of Lucas et al. were taken on a system in which the bed was electrically heated, as also was the gas before it entered the bed. They found the bed temperature to be uniform within 3 K. These workers have reported that for the same system, as the temperature is changed  $\epsilon_{mf}$  undergoes a nonmonotonic variation, and the nature of this variation depends on the size of the particle. They found that for small particles  $\epsilon_{mf}$  increases with temperature and tends to become constant at higher temperatures; for larger particles  $\epsilon_{mf}$  first decreases with increasing temperature, passes through a minimum, and then increases with further increase in temperature; and for still larger particles  $\epsilon_{mf}$  remains constant with increasing bed temperature. No precise limits for the particle sizes are specified. Mathur and Saxena (1985) have analyzed these observations in the perspective of the powder classification scheme of Saxena and Ganzha (1984). They infer that for systems corresponding to groups I and III of the powder classification scheme  $\epsilon_{mf}$  may be regarded as a constant quantity, but for systems belonging to groups IIA and IIB  $\epsilon_{mf}$  will change with change in  $Ar$ . Physically this is explained by the changing nature of the flow field around the particles, which in turn influences the nature of interparticle forces, and hence  $\epsilon_{mf}$ .

The experimentally observed and physically explained variation of  $\epsilon_{mf}$  with temperature and pressure mentioned above raises questions about the proposed methodology of Yang et al., which is based on the concept of invariant  $\epsilon_{mf}$ . On the other hand, the observed success of the methodology as judged on the basis of available experimental data stresses its potential overall ability to predict  $U_{mf}$ . The limitation of assuming the particles to be spherical in the computation scheme is not capable of reconciling this discrepancy as the two are not equivalent assumptions. It may be pointed out that Chitester et al. (1984) in their measured  $\epsilon_{mf}$  values for different particles at ambient temperature

and at pressures in the range 101 kPa to 6.3 MPa found it to be feebly dependent on pressure:  $\epsilon_{mf}$  increased slowly with pressure and reached a constant value at higher pressures. We have endeavored to outline these facts in the hope that continuing experimental efforts will provide clues for these experimental findings.

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### Notation

$Ar$  = Archimedes number,  $gd_p^3\rho_g(\rho_s - \rho_g)/\mu^2$   
 $C_D$  = drag coefficient for a single particle  
 $d_p$  = average particle diameter, m  
 $g$  = acceleration due to gravity, m/s<sup>2</sup>  
 $Re$  = Reynolds number,  $Ud_p\rho_g/\mu$   
 $U$  = superficial gas fluidizing velocity, m/s

### Greek letters

$\epsilon$  = bed voidage  
 $\mu$  = gas viscosity, kg/ms  
 $\rho_g$  = gas density, kg/m<sup>3</sup>  
 $\rho_s$  = solids density, kg/m<sup>3</sup>

### Subscripts

$\epsilon$  = modified for multiparticle systems  
 $mf$  = corresponding to the value at minimum fluidization

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